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13MCA12

First Semester MCA Degree Examination, Dec.2014/Jan.2015
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Show that the implication $\neg (p \rightarrow q) \rightarrow \neg q$ is a tautology. **(06 Marks)**

b. For primitive statements p, q show that $(p \vee q) \wedge \neg (\neg p \wedge q) \Leftrightarrow p$. **(07 Marks)**

c. For all integers k and *l*, if k, *l* are both odd, then prove that k + *l* is even. **(07 Marks)**

2 a. Negate and simplify giving reasons: $\neg[\neg[(p \vee q) \wedge r] \vee \neg q]$. **(06 Marks)**

b. Validate the argument giving reasons $[(p \rightarrow r) \wedge (r \rightarrow s) \wedge (t \vee \neg s) \wedge (\neg t \vee u) \wedge \neg u]$. **(08 Marks)**

c. Negate the following statement: $\forall x \exists y[(p(x, y) \wedge q(x, y) \rightarrow r(x, y)]$. **(06 Marks)**

3 a. Define union, intersection and symmetric difference of two sets A and B. Give an example to each. **(06 Marks)**

b. State and prove the distributive law of $\cap$ over $\cup$. **(07 Marks)**

c. In a survey of 120 passengers, an airline found that 48 enjoyed wine with their meals, 78 enjoyed mixed drinks and 66 enjoyed iced tea. In addition, 36 enjoyed any given pair of these beverages and 24 passengers enjoyed them all. If two passenger's are selected at random from the survey sample of 120, what is the probability that

i) (Event A) they both want only iced tea with their meals?

ii) (Event B) they both enjoy exactly two of the three beverage offerings? **(07 Marks)**

4 a. Show that $2^n > n^2$ wherever 'n' is a positive integer greater than 4. **(07 Marks)**

b. Prove that $1.2 + 2.3 + 3.4 + \ldots\ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$. **(06 Marks)**

c. For any $n \in z^+$, prove that the integers 8n + 3 and 5n + 2 are relatively prime. **(07 Marks)**

5 a. Let A = {a, b, c, d, e, f, g, h} and B = {1, 2, 3, 4, 5}.

i) How many elements are there in P(A × B)?

ii) Generalize the result in (i). **(06 Marks)**

b. What is the injection function? Verify whether the following function are one-to-one or not:

i) $f : R \rightarrow R$, $f(x) = 3x + 7$, $\forall x \in R$

ii) $g : R \rightarrow R$, $g(x) = x^4 - x$, $\forall x \in R$. **(07 Marks)**

c. If A = {1, 2, 3, 4, 5}, give an example of a relation 'R' on 'A' that is

i) Reflexive and symmetric but not transitive.

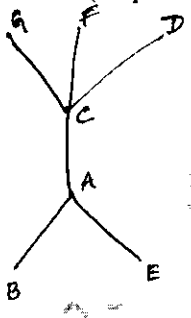
ii) Reflexive and transitive but not symmetric.

iii) Symmetric and transitive but not reflexive. **(07 Marks)**

6 a. Let A = {a, b, c, d, e, f}. Determine the relation $R \subseteq A \times A$ and draw directed graph G associated with 'R' for the following relational matrices: **(06 Marks)**

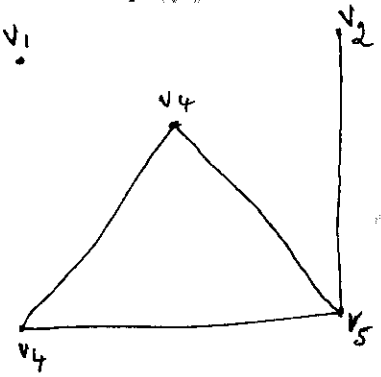
$$M(R) = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

b. Topologically sort the following partial order (stepwise): **(07 Marks)**



c. Let A = {1, 2, 3, 12}.
   i) Construct the matrix tables for glb{x, y} and lub{x, y} in [A, 1].
   ii) Construct the meet-join table.
   iii) Is [A, 1] a lattice? **(07 Marks)**

7 a. Define complement of a graph. Write the complement of the following graph G. **(06 Marks)**



b. Prove that the graphs $K_{3,3}$ and K_5 are nonplanar. **(07 Marks)**

c. Define the following and give one example for each:
   i) A line graph; ii) A bipartite graph; iii) A clique. **(07 Marks)**

8 a. Define chromatic number of a graph, prove that "the chromatic number of any connected bipartite graph with atleast two vertices is 2". **(07 Marks)**

b. Define a tree. Prove that the number of edges in a tree with 'n' nodes is n-1. **(06 Marks)**

c. Write the Merge sort algorithm. **(07 Marks)**

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